Four Tables from the Temple Library of Nippur:

A Source for 'Plato's Number'
in relation to the
Quantification of Babylonian Tone Numbers

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Abstract

The four cuneiform texts discussed here were originally published by Hilprecht in his twentieth volume of the Babylonian Expedition of the University of Pennsylvania, in 1906[1]. They came from the temple library of Nippur and were part of some 7000 texts and fragments Hilprecht catalogued. The volume lists the texts as numbers 20, Rev.; 21, Rev.; 22, Obv. and 24, Rev. They are dated about 2200 BC. (Tables I-IV).[2]

Hilprecht refers to the texts as tables of multiplication and division. However, the purpose for these peculiar operations was not fully understood, mainly for the reason that in the early twentieth century, texts of theory, specifically UET VII 74 and 126; CBS 1766 and 10996, had not yet been satisfactorily interpreted. Although Hilprecht saw similarities with 'Plato's number' as laid out in Republic, Book VIII[3], He did not perceive

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[1] Hilprecht, H.V., The Babylonian Expedition of the University of Philadelphia – Series A: Cuneiform Texts, (Hilprecht, ed.) Volume XX, Part I *Published by the Department of Archaeology, University of Pennsylvania* (1906)

[2] Hilprecht, H.V., The Babylonian..., Pls 10; 11; 12; 14; CBM 11340 + 11402; 11368; 11902; 11097.

[3] Adam, J., The Republic of Plato, Book VIII, appendices, pp. 264-318. (Cambridge, 1902)

that the lacunae of certain numbers, in addition to other mathematical purposes, were also the consequence of their relation to music theory.

The thesis is constructed from the premise that these omissions, in our present knowledge of Babylonian mathematics, cannot suit, in practice, any other systems such are enunciated, principally, in UET VII 74; UET VII 126; CBS 1766 and CBS 10996[4].

The texts

As judiciously observed, Hilprecht noted that the four tablets shared three particular features:

1. When several multiplication tables are written together, the highest number begins the series.
2. The numbers multiplied are not consecutive, but are often separated from each other by comparatively large intervals.
3. Besides 3 and 5, no indivisible number or its multiple is multiplied. Note the absence of 7; 11; 13; 14; 17; 19; 21; 22; 23; 26; 28; 29; 31; 33; 34; 35; 37; 38; 39; 41; 42; 43; 44; 46; 47; 49; 51; 52; 53; 55; 56; 57; 58; 59; 61; 62; 63; 65; 66; 67; 68; 69; 70; 71; 73; 74; 75; 76; 77; 78 and 79.

Out of 81 numbers, 51 are ommitted leaving, significantly, 30 numbers.

The four fragmentary texts from which Hilprecht reconstructs one whole table, throw considerable light on the whole question of their musicological significance:

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[4] Dumbrill, R.J., The Archaeomusicology of the Ancient Near East (Oxford 2005) pp. 11-110.

1	8.640.000 *A-AN*[5]	25	518.000
2	6.480.000	27	480.000
3	4.320.000	30	432.000
4	3.240.000	32	405.000
5	2.592.000	36	360.000
6	2.160.000	40	324.000
8	1.620.000	45	288.000
9	1.440.000	48	270.000
10	1.296.000	50	259.000
12	1.080.000	54	240.000
15	864.000	60	216.000
16	810.000	64	202.500
18	720.000	72	180.000
20	648.000	[80	162.000]
24	540.000	[81	160.000]

They all have 12,960,000 (= 60^4 and 3600^2) as their dividend.

Hilprecht attempted at finding a reason for this number and found some answers, as I have mentioned above, with Plato's Republic, Book VIII, 546, B-D.

What Plato says:

It makes little doubt that Plato attempted at what proved to be a very successful numerological-mythological manipulation of a much older Babylonian story which he kept to himself, for his own purposes. As basis for all of his calculations, leading up to the the construction of 'his number', he uses the Pythagorean triangle. The right-angled triangle in question is one in which both sides are 3 and 4 with a hypotenuse of 5, naturally. Now, the addition of the cubes of each of the sides gives: [3^3+ 4^3 + 5^3 = 216] which is Plato's number for his 'period' being women's shortest gestation time, counted in days - 270 days being the period for 9 months' gestation. The surface of the triangle is [3 x 4 ÷ 2 = 6]. The multiplication of the three sides of the triangle gives [3 x 4 x 5 = 60]. Now 60^2 = 3600; 60^3 = 216.000 and 60^4 = 12.960.000. This is the famous famous number.

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[5] Hilprecht rightly notes that: '*They all read: 1 - 8.640.000 A-AN. The quotient being 2/3 of 12.960.000, we should rather expect '1 ½' instead of '1' as its divisor, for 12.960.000 divided by 3/2 = 12.640.000 x 12.960.000 x 2/3.*'

Plato's numerics are not very complex. He wants to relate all to 'his number'. This is a concept very similar to the thinking behind modern 'string theory': the theory to explain all others.

Plato says that there are four leading varieties of states: timarchy which is made up of the Cretan and Laconian states; oligarchy; democracy and tyranny. The character of the states is determined by their individuals. There are five leading types of characters: the aristocratic; the timarchical; the oligarchical; the democratical, and the tyrannical. Timarchy arises from aristocracy through constitutional changes originating from disagreements between the ruling classes. The muses are asked to tell how discord started. They say that everything has a beginning and an end; even the ideal city will perish. However, the cause of decay, which does not belong in the ideal city itself, must come from outside. But always, degeneration starts by badly planned marriages and births. Things have not changed much.

As I said above, Plato makes up a number which comes from the duration of a woman's shortest period of gestation which he says is 216 days. The number with which he comes is 12.960.000. He defines it as the '*Lord of Better and Worse Births*'. In fact it is the numeric form of a 'G*reat Law controlling the Universe*', or the answer to all. The divine being whom Plato calls '*The World brought from Chaos to Order*' has a number defined by the duration of its creation. The human being has a period of gestation defined by the first number, after the unit, in which root and square increase: This is the multiplication. It comprises three elements: length; width and thickness. It has four limitations. These are the points by which the three dimensions are determined, rendering all things tangible and comparable: 3 x 3^2 (= 3^3 = 27) + 4 x 4^2 (= 4^3 = 64) + 5 x 5^2 (= 5^3= 125) = 216); two of which (*numbers to be cubed,*) namely 4, 3 (*forming two of the three sides of the Pythagorean triangle*), coupled with 5 (*by multiplication*, *i.e.,* 4 x 3 x 5 = 60) furnish two harmonies when three times increased (*i.e., three times multiplied by itself, therefore* 60 x 60 x 60 x 60 = 12,960,000) - the one equal an equal number of times (*i.e., a square number*), so many (= 36) times 100 (= 3600^2), the other of equal length one way, but oblong: on the one side, of one hundred squares of rational diameters of five (*i.e., the nearest rational number to the real diameter of a square whose side is five, i.e.*, to $\sqrt{50}$ = 7; for $\sqrt{49}$ = 7), diminished by one each (*i.e.,* (49 [= 7^2] x 100) – (1 x 100) = 4900 - 100 = 4800), or if from irrational diameters (*of five*), diminished by two (*each*) (*i.e.* 50 x 100 – 2 x 100 = 5000 - 200 = 4800), on the other hand, of 100 cubes of 3 (*i.e.,* 100 x 3^3 = 2700; *the second harmony is therefore* 4800 x 2700).

The two harmonies Plato gives with 60 x 60 x 60 x 60 (*i.e.,* 3600^2 and 4800 x 2700, *both* = 12,960,000) represent two recurrent aeons in the life of the universe, in which the world waxes and wanes alternately. The harmony 3600^2 measuring the cycle of uniformity, and the harmony 4800 x 2700 is the cycle of dissimilarity. It goes without saying that the number 3600 rests upon the Babylonian sexagesimal system. From the fact that Plato constructs his number from the shortest period of gestation, it may be inferred that it stands in a mathematical relation to other numbers. Since the smaller number refers to days, it is safe to interpret 12,960,000 also as days. Now 12,960,000 days, expressed in years, 360 days counted per year, are equal to 36,000 years. We know from Berossus, writing in the third century BC, that a period of 36,000 years, is called the '*Great Platonic Year*', in early astronomical treatises, and that this was the duration of a

Babylonian cycle. We also have shown that multiplication and division tables from the temple libraries of Nippur are based on 12,960,000. This coincidence cannot be accidental. We must come to the conclusion that Plato borrowed his famous number and the whole idea straight from the Babylonian model.

The Platonic number, 12,960,000, measuring a period in the lifetime of the Earth, is the '*Lord of Better and Worse Births.*' In what sense this number, which is the square of the highest Babylonian figure determines good and bad births, has been explained in various ways. In the early stages of the cycle of 36,000 years, before disintegration and dissimilarity, nature produced better children than later. This was because the universe was growing worse. But this interpretation is not sufficient. Evidently Plato wants to bring out the double idea firstly that through the ignoring of a fundamental law of the universe at some early time, disagreement arose, followed by a subsequent degeneration of the whole human race; and secondly that the same fundamental law still governs the universe, and that its breach at any time is accompanied by the same result. Although it is true that births at a later stage of our cycle of 36,000 years are comparatively worse than those at an earlier period, the former are by no means of equal value. On the contrary, in Plato's words: '*Whenever our guardians promote marriages inopportunely, the offsprings cannot be well provided.*', necessarily implies that whenever the guardians observe these principles, the children born will be healthy and prosperous. Thus good births are not confined to an earlier stage of the life of the universe and bad ones to a later one, but that good and bad births may occur at all times within our cycle. Whether a birth is good or bad is determined by the number 12,960,000, which for this very reason is called '*Lord of Better and Worse Births.*' The meaning of Plato's words, therefore, can only be that in order to be a good birth, the birth of a child must stand in a certain relation to 12,960,000, as the arithmetical expression of a fundamental law of the universe.

The answer as to how this number can influence the birth and future of a child lies within the Babylonian concept of the world. The universe and everything within, whether great or small, is created and ruled by the same fundamental law. The same forces and principles which rule the world's macrocosm, also rule man's microcosm. It is the purpose of astronomy to prove this harmony, and to determine the laws of invariance which rule the universe. The sky, from which the gods reveal themselves, is the great book, the '*Writing of Heaven*', in which '*they*' have written the whole story of heaven and earth, its past, present and future. The astronomer studies and deciphers this divine writing, the astrologer interprets its meaning with regard to the life and affairs of man.

According to this concept, all on earth, including the state and the family; the temples and cities of Babylon, are fashioned from heavenly patterns; and all human knowledge and science, including mathematics and astronomy, even the number itself, the division of the circle into 360 degrees, the calendar, the system of measures and weights, and the laws of music are all of divine origin. Seven, eleven and thirteen are not divisors of 12,960,000, and to this day are still regarded as unlucky numbers.

The hypothesis

Whatever Plato's justification for 'his number', it provides with a satisfactory purpose for the Nippur texts.

The Pythagorean right-angled triangle has sides which measure 3, 4 and 5. Therefore they have 3:4:5 as ratios between them:. The ratio of 5:6 is made up from the doubling of side 3 in relation to the hypotenuse. Ratios of 1:2 and 2:3 arise from the halving of 4. Thus we have 1:2; 2:3; 3:4; 4:5 and 5:6. These ratios correspond to the first divisors in Hilprecht's reconstruction. However, the divisor '1' should relate to 12.960.000, and not to 8.640.000 whose divisor should be $1^{1/2}$. Hilprecht was concerned by this discrepancy and writes: *'I am unable to explain this strange phenomenon. Possibly we have to regard it as an abbreviated expression well understood by the Babylonians'.* I do not see, either, any reason for this other than an irrational one, or, as Crickmore puts it to me, in a recent communication: '*...could line one, for example, be a concession to practical musicians, who are not generally noted for their mathematical expertise? Or, could it be a reminder for theoretical musicians that the whole of these tables can have an application in a musical context? Or is it simply the scribe's dedication of the table to Ea, the 'god' of music?'* Indeed, if we read the sign as *šuššu*, = 60, Anu's number, referring to the musical string of 60 *ubanātu*, then 60 x 2/3 = 40 which is Ea's number.

The Neo-Babylonian text UET VII, 126[6], published by the late Professor O.R. Gurney, shows that in l.4[7] a fourth-string is listed, bilingually, as Sumerian 'sa.4.tur' = 'fourth, small-string', and Akkadian *'a-ba-nu-[ú]'* = 'Ea-created'. It is the only string, with a god relationship, mentioned in the text. The pairing of Ea with this fourth string whilst the god is usually associated with number 40[8] attracted my attention. Might there have been an earlier numbering of the gods where only the leading deities were listed as: 6 for *Anu*; 5 for *Enlil*; 4 for *Ea*; 3 for *Sin* and 2 for *Šamaš,* whilst they are usually known as 60; 50; 40; 30 and 20, respectively. The ratios between them would be: 6:5; 5:4; 4:3; and 3:2. In the sexagesimal musical scale, the ratio of 6:5 is the minor third; the ratio of 5:4 is the major third; 4:3 is the fourth and 3:2 the fifth. But of course, their decimalisation produces the same ratios. These constitute the essential intervals of the Babylonian musical system as we know it from CBS 10996 where they are clearly defined although restricted within a heptachordal system which accounts, as a consequence, for thirds inversed into sixths and and fifths inversed to fourths. The text mainly names intervals in function of their position in the following pattern: 1-5/7-5; 2-6/8-6; 3-7/9-7, etc. (in fact the pattern being restricted to the heptachord reads: 1-5/7-5; 2-6/1-6, etc.) The divisors for these intervals are 2-3, for the interval of the fifth and 6-5 and 5-4 for minor and major thirds, respectively.

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[6] Best identified as a Late Babylonian manuscript of tablet XXXII of the text *Nabnītu.* See Finkel, I.L., *Materials for the Sumerian Lexicon (MSL)* XVI, 251; Wulstan, D., *The Tuning of the Babylonian Harp,* IRAQ XXX, (1968), 215-228; *The Earliest Musical Notation,* MUSIC AND LETTERS 52, (1971), 365-382; Duchesne-Guillemin, M., *A Hurrian Musical Score from Ugarit: The Discovery of Mesopotamian Music,* SANE, Vol. 2, Fasc. 2, (Malibu 1984) 5-24; *Survivance Orientale dans la désignation des Cordes de la Lyre en Grèce,* SYRIA 44, (1967), 233-246; *A l'aube de la Théorie Musicale: Concordance de Trois Tablettes Babyloniennes,* REVUE DE MUSICOLOGIE 52, (1966), 147-162; Crocker, R., and Kilmer, A.D., *The Fragmentary Text from Nippur,* IRAQ XLVI, (1984), 81-85; *A Music Tablet from Sippar,* IRAQ XLVI, Part 2, (1984), 69-79; Vitale, R., *La Musique Suméro-Accadienne,* UGARIT-FORSCHUNGEN, (1982), 241-263; West, M.L., *The Babylonian Musical Notation and the Hurrian Musical Texts,* MUSIC AND LETTERS, 75/4, (1993), 161-179; Dumbrill, R.J., Babylonian Theonumerics and Scale Systems, JAC, Volume 22, 2007.

[7] Having read the manuscript of the present paper, Prof. Andrew George brought to my attention that the 4th line of the Akkadian column of Professor O.R. Gurney's VIIth tome of the Ur Excavation Texts (1973), Pl. LX, only shows '*a*' and not '*a-ba-nu-[ú]*' (Kilmer), or A.DÙ (Finkel), I wrote to Gurney who replied to me that the tablet, now in Baghdad, read *a-ba-nu-[ú]* but that inadvertently had left *ba-nu-[ú]* out. The error is corrected in IRAQ XL, VI, 82, fn. 1.

[8] Röllig, W., REALLEXIKON DER ASSYRIOLOGIE *sub* Götterzahlen, p. 499-500; Livingstone, A., *Mystical and Mythological Explanatory works of Assyrian and Babylonian Scholars*, (Oxford, 1986), , 30-49; Parpola, S., *The Assyrian Tree of Life: Tracing the Origins of Jewish Monotheism and Greek Philosophy, JNES,* 52, n. 3, (1993), 182-184, fn. 86-89.

UET VII, 126 lists strings in a particular manner showed below (Fig.1) which explains a tuning method devised for an enneatonic system, a forerunner for the heptatonic model. The tuning starts from the central note which is the tonic-central note from which fifths, rising and falling, make up the enneatonic span. Fourths rise and fall from the extremities. Thirds, minor and major appear as a consequence. It follows that the range of divisors resulting from this method is placed between the Nippur numbers 36 and 80: 36; 40; 45; 48; 54; 60; 64; 72; 80. Not only all of these numbers are divisors in Hilprecht's reconstruction, but the missing numbers in his table are also absent from the pattern in UET VII 126. It is obvious that this cannot be taken as purely coincidental. Significantly, UET VII 126 has 80 as its highest number. The smallest interval produced by the divisors also ends Hilprecht's reconstruction, with the ratio of 80:81. This is the syntonic comma also known as Ptolemaic comma or comma of Didymus, and measures 22 cents.

CBS 1766[9] produces a generative scale which finds its place within the divisors 27 and 48 as: 27; 30; 32; 36; 40; 45 and 48 whilst the enneatonic system in UET VII 74 places its tones within the range of 25 to 50 with the following divisors: 25; 27; 30; 32; 36; 40; 45; 48 and 50. Here again, these scales use all of the numbers present in Hilprecht's table to the exclusion of any other.

Conclusion

None of the music theory text discovered so far have produced any tone number which is not replicated in Hilprecht's table where each number corresponds to a pitch known to the Babylonians. It is very likely that because of their particular structure, the Nippur texts were in fact tables, additionally to what might have been their original purpose, if any, listing the full range of tone numbers known to Babylonian music theory. Now the other columns where the numbers are in the form of n^2; n^3 and n^4 provide with the quantification of more sophisticated intervals such as will be revealed in centuries to follow, notably with Plato's 'two harmonies' which is precisely located in the n^3, in the range 2700; 3000; 3375; 3600; 4000; 4320; 4500 and 4800. In the 6th century AD, Boethius, as a keen Neo-Platonist, uses numbers in the Nippur tables, and so do later theoreticians such as Farabi, Avicenna, Safi ad-din, Salinas and many more. To this day, the integrality of the figures in the Nippur tables still constitute the basis for the study of music theory (Fig. 2).

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[9] Waerzeggers, C., and Siebes, R., '*An alternative Interpretation of the seven-pointed star on CBS 1766 (Horowitz, JANES 30)* N.A.B.U., 2007, no.2, June, pp. 43-45; Dumbrill, R.J., forthcoming in IRAQ?; Crickmore, L., forthcoming.

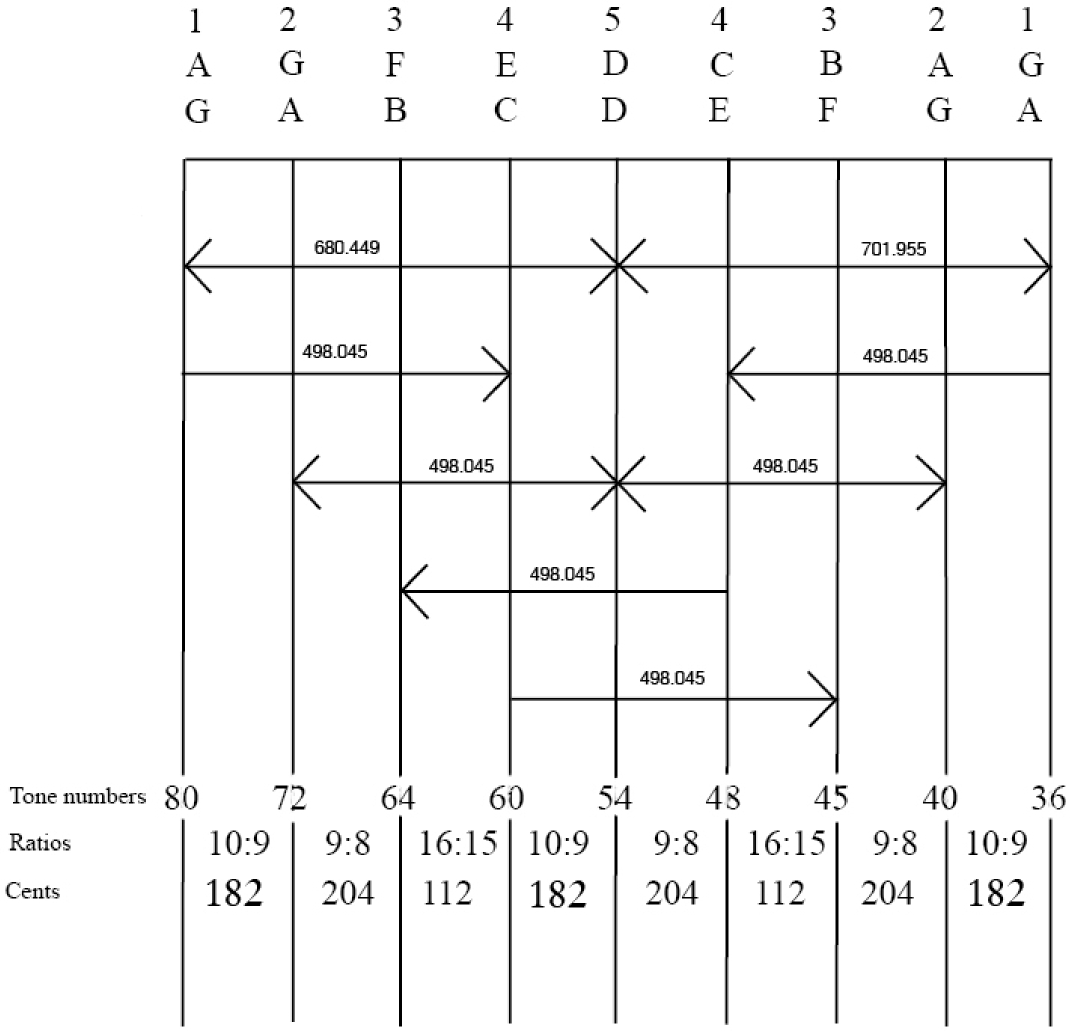
1 2 3 4 5 4 3 2 1
A G F E D C B A G
G A B C D E F G A
680.449
701.955
498.045
498.045
498.045
498.045
498.045
498.045
Tone numbers 80 72 64 60 54 48 45 40 36
Ratios 10:9 9:8 16:15 10:9 9:8 16:15 9:8 10:9
Cents 182 204 112 182 204 112 204 182

Figure 1. Diagrammatic rendition of UET VII 126.

Nippur 60^4	Divisor	↑Pitch	↓Pitch	Cents	Ratio
12.960.000	[1]	F	B		
				701.954	2:3
8.640.000	[$1^{1/2}$]	C	E		
				498.045	3:4
6.480.000	**2**	F	B		
				701.954	2:3
4.320.000	**3**	C	E		
				498.045	3:4
3.240.000	**4**	F	B		
				386.314	4:5
2.592.000	**5**	A	G		
				315.641	5:6
2.560.000	**6**	C	E		
				498.045	8:6 =3:4
1.620.000	**8**	F	B		
				203.910	8:9
1.444.000	**9**	G	A		
				182.404	9:10
1.296.000	**10**	A	G		
				315.641	10:12 = 5:6
1.080.000	12	C	E		
				386.314	12:15 = 4:5
864.000	**15**	E	C		
				111.731	15:16
810.000	**16**	F	B		
				203.910	16:18 = 8:9
720.000	**18**	G	A		
				182.404	18:20 = 9:10
648.000	**20**	A	G		

				315.641	20: 24 = 5:6
540.000	**24**	C	E		
				70.672	24:25
518.000	**25**	C#	Eb		
				133.237	25:27
480.000	**27**	D	D		
				182.404	27:30 = 9:10
432.000	**30**	E	C		
				111.731	30:32 = 15:16
405.000	32	F	B		
				203.910	32:34 = 8:9
360.000	**36**	G	A		
				182.404	36:40 = 9:10
324.000	**40**	A	G		
				203.910	40:45 = 8:9
288.000	**45**	B	F		
				111.731	45:48 = 15:16
270.000	**48**	C	E		
				70.672	48:50 = 24:25
259.000	50	C#	Eb		
				133.237	50:54 = 25:27

				182.404	54:60 = 9:10
216.000	**60**	E	C		
				111.731	60:64 = 15:16
202.500	**64**	F		B	
				203.910	64:72 = 8:9
180.000	**72**	G	A		
				182.404	72:80 = 9:10
62.000	**80**	A	G		
				21.506	80:81
160.000	**81**	A*	G*		
144.280	**90**	B	F		

Figure 2. Musicological rendition of Hilprecht's table

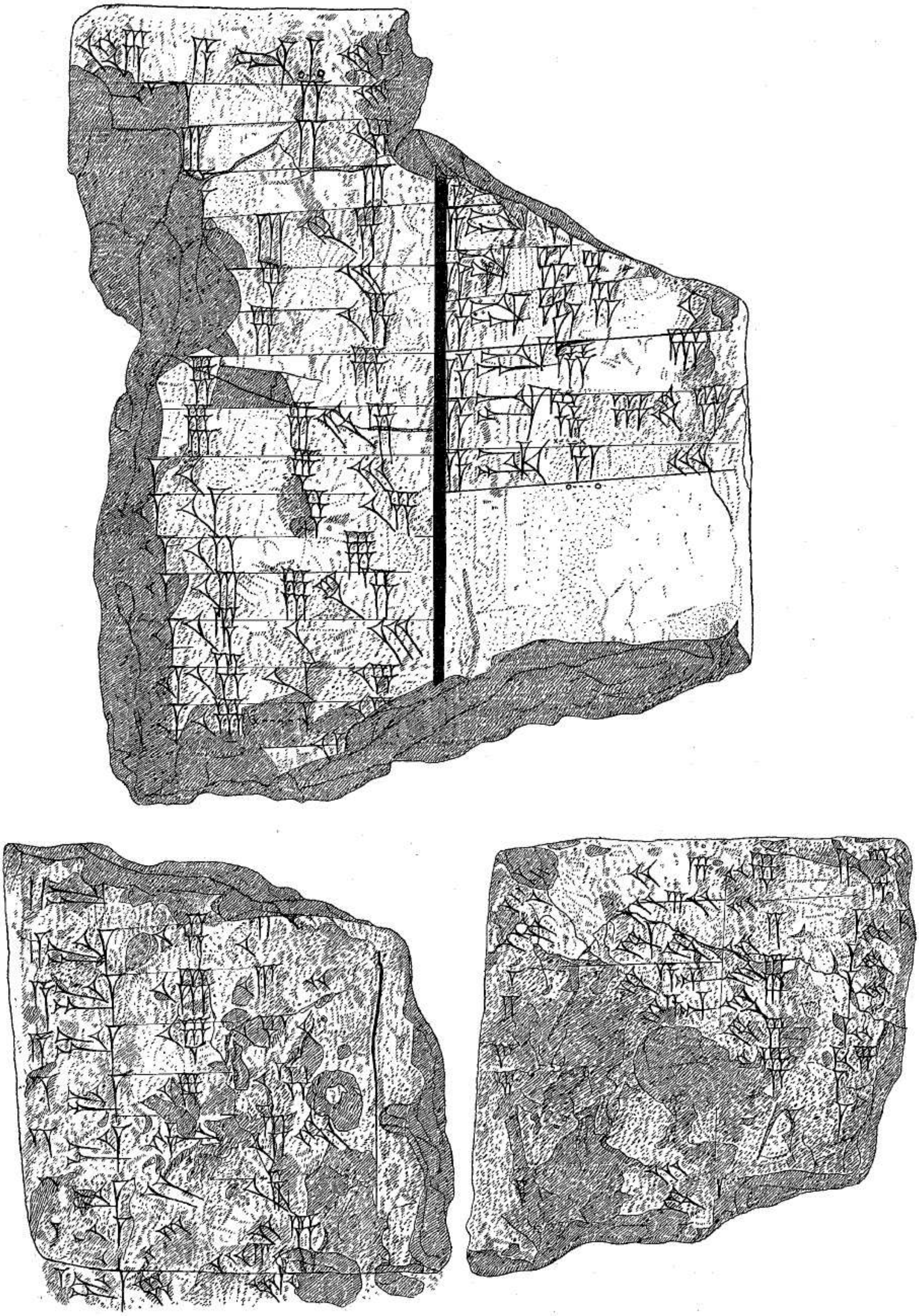

Table I. 20 Obv. CBM 11340 + 11402; 21 Obv. and Rev. CBM 11368

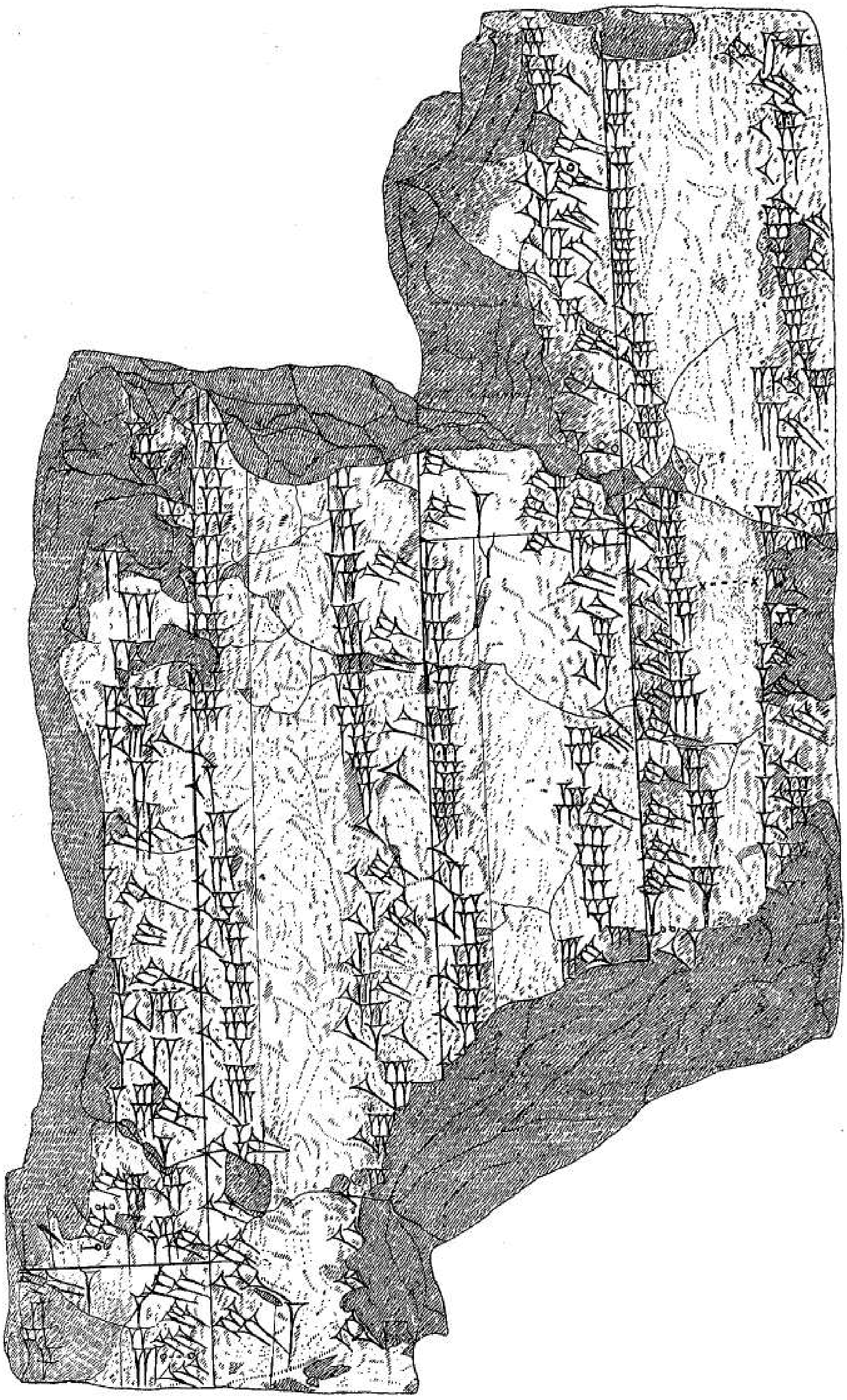

Table II. 20 Rev. CBM 11340 + 11402

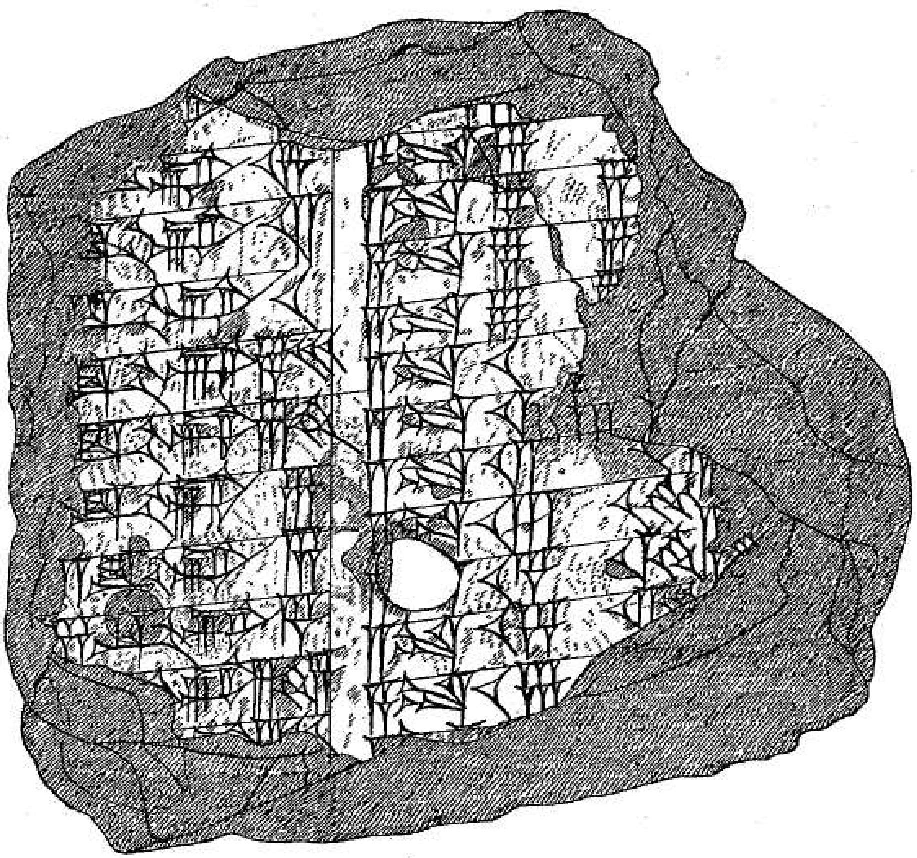

Table III. 22 Obv. CBM 11902

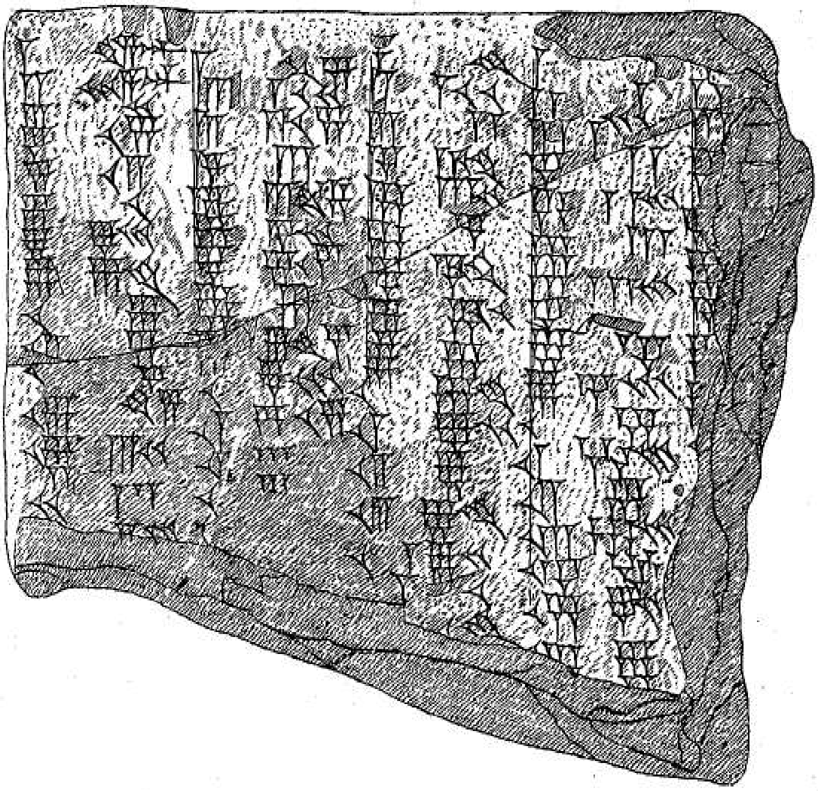

Table IV. 24 Rev. CBM 11097